

Math 115

Fall 2018

Lecture 15

$$? a^2 + b^2 = c^2 ?$$

$$y = mx + b \quad ? \quad d = rt$$

Feb 19-8:47 AM

Exponential Rules:

$$1) \quad x^n = \underbrace{x \cdot x \cdot x \cdot \dots \cdot x}$$

$x \rightarrow$ Base n times

$n \rightarrow$ Exponent or Power

$$\text{ex: } x^4 = x \cdot x \cdot x \cdot x$$

$$(2x)^3 = (2x) \cdot (2x) \cdot (2x)$$

$$(-5x^2y^3)^5 = \underbrace{(-5x^2y^3) \cdot (-5x^2y^3) \cdot \dots \cdot (-5x^2y^3)}_{5 \text{ times}}$$

$$\left(\frac{-2}{3}\right)^2 = \left(\frac{-2}{3}\right) \cdot \left(\frac{-2}{3}\right) \quad 5 \text{ times}$$

Nov 15-6:08 AM

$$2) x^1 = x$$

$$5^1 = 5$$

$$(-12)^1 = -12$$

$$(3x^5)^1 = 3x^5$$

$$\left(\frac{-x^4}{y^6}\right)^1 = \frac{-x^4}{y^6}$$

$$3) x^0 = 1, x \neq 0$$

$$5^0 = 1$$

$$(-12)^0 = 1$$

$$\left(\frac{3}{7}\right)^0 = 1$$

$$(23x^4y^5)^0 = 1; \begin{matrix} x \neq 0 \\ y \neq 0 \end{matrix}$$

Nov 15-6:12 AM

$$4) x^m \cdot x^n = x^{m+n}$$

$$x^3 \cdot x^7 = x^{3+7} = x^{10}$$

$$x^8 \cdot x^{11} \cdot x^1 = x^{8+11+1} = x^{20}$$

$$(3x^2)^6 \cdot (3x^2)^9 = (3x^2)^{6+9} = (3x^2)^{15}$$

$$\begin{aligned} (-7x^3y^6)^5 \cdot (-7x^3y^6)^8 \cdot (-7x^3y^6)^7 \\ = (-7x^3y^6)^{5+8+7} = (-7x^3y^6)^{20} \end{aligned}$$

Nov 15-6:14 AM

$$5) (x^m)^n = x^{m \cdot n}$$

$$(x^4)^6 = x^{4 \cdot 6} = x^{24}$$

$$(x^7)^{10} = x^{7 \cdot 10} = x^{70}$$

$$\begin{aligned} (x^8)^4 \cdot (x^4)^3 &= x^{8 \cdot 4} \cdot x^{4 \cdot 3} \\ &= x^{32} \cdot x^{12} = x^{32+12} = \boxed{x^{44}} \end{aligned}$$

$$(x^7)^3 \cdot (x^{-5})^4 = x^{21} \cdot x^{-20} = x^{21+(-20)} = x^1 = \boxed{x}$$

Nov 15-6:18 AM

$$6) \frac{x^m}{x^n} = x^{m-n}$$

$$\frac{x^8}{x^3} = x^{8-3} = \boxed{x^5}$$

$$\frac{x^{11}}{(x^2)^5} = \frac{x^{11}}{x^{10}} = x^{11-10} = x^1 = \boxed{x}$$

$$\frac{(x^6)^5}{(x^{10})^3} = \frac{x^{6 \cdot 5}}{x^{10 \cdot 3}} = \frac{x^{30}}{x^{30}} = x^{30-30} = x^0 = \boxed{1}$$

$x \neq 0$

Nov 15-6:24 AM

<p>Simplify</p> $\frac{(x^7)^2 \cdot x^{10}}{(x^3)^8}$ $= \frac{x^{14} \cdot x^{10}}{x^{24}}$ $= \frac{x^{24}}{x^{24}}$ $= x^{24-24} = x^0 = \boxed{1} \quad x \neq 0$	<p>Simplify</p> $\frac{(x^6)^6}{(x^{-12})^3}$ $= \frac{x^{36}}{x^{-36}}$ $= x^{36 - (-36)}$ $= x^{36+36} = \boxed{x^{72}}$
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Nov 15-6:28 AM

⑦ $(xy)^n = x^n y^n$

$$(2x)^5 = 2^5 x^5 = \boxed{32x^5}$$

$$(-4x^2)^3 = (-4)^3 (x^2)^3 = \boxed{-64x^6}$$

$$(-3x^7y^{10})^4 = (-3)^4 (x^7)^4 (y^{10})^4$$

$(-)^{\text{odd}} = -$

$$= + 81x^{28}y^{40} = \boxed{81x^{28}y^{40}}$$

$(-)^{\text{even}} = +$

Nov 15-6:33 AM

Simplify:

$$\begin{aligned} & (-2x^6)^3 \cdot (x^3)^4 \\ &= (-2)^3 \cdot (x^6)^3 \cdot (x^3)^4 \\ &= -8x^{18} \cdot x^{12} \\ &= \boxed{-8x^{30}} \end{aligned}$$

Simplify:

$$\begin{aligned} & (-5x^8)^2 \cdot (4x^3)^3 \\ &= (-5)^2 (x^8)^2 \cdot 4^3 \cdot (x^3)^3 \\ &= 25 \cdot x^{16} \cdot 64 \cdot x^9 \\ &= 25 \cdot 64 \cdot x^{16} \cdot x^9 \\ &= \boxed{1600x^{25}} \end{aligned}$$

Nov 15-6:37 AM

$$8) \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

$$\left(\frac{x^4}{2}\right)^5 = \frac{(x^4)^5}{2^5} = \boxed{\frac{x^{20}}{32}}$$

$$\left(\frac{-4x^5}{y^6}\right)^3 = \frac{(-4x^5)^3}{(y^6)^3} = \frac{(-4)^3 (x^5)^3}{(y^6)^3} = \boxed{\frac{-64x^{15}}{y^{18}}}$$

Nov 15-6:46 AM

Simplify:

$$\begin{aligned} \left(\frac{-3x^{10}}{5y^6} \right)^3 &= \frac{(-3x^{10})^3}{(5y^6)^3} \\ &= \frac{(-3)^3 (x^{10})^3}{5^3 (y^6)^3} \\ &= \boxed{\frac{-27x^{30}}{125y^{18}}} \end{aligned}$$

Nov 15-6:49 AM

$$9) \quad x^{-n} = \frac{1}{x^n}$$

$$x^{-2} = \frac{1}{x^2}$$

$$(x^5)^{-4} = x^{5 \cdot (-4)}$$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$

$$= x^{-20}$$

$$= \frac{1}{x^{20}}$$

$$(5x^6)^{-3} = \frac{1}{(5x^6)^3} = \frac{1}{5^3 (x^6)^3} = \boxed{\frac{1}{125x^{18}}}$$

Nov 15-6:53 AM

Simplify:

$$\begin{aligned} (x^7)^{-4} \cdot (x^{-3})^4 &= x^{-28} \cdot x^{-12} \\ &= x^{-28+(-12)} \\ &= x^{-40} = \boxed{\frac{1}{x^{40}}} \end{aligned}$$

Simplify

$$\begin{aligned} \frac{(x^{-6})^5 \cdot x^{-10}}{(x^{-8})^{-5}} &= \frac{x^{-30} \cdot x^{-10}}{x^{40}} = \frac{x^{-40}}{x^{40}} \\ &= x^{-40-40} = x^{-80} = \boxed{\frac{1}{x^{80}}} \end{aligned}$$

Nov 15-6:56 AM

$$\textcircled{10} \frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$$

$$\frac{x^{-3}}{y^{-7}} = \boxed{\frac{y^7}{x^3}}$$

$$\frac{(x^8)^{-2}}{(y^{-10})^3} = \frac{x^{-16}}{y^{-30}} = \boxed{\frac{y^{30}}{x^{16}}}$$

$$\frac{x^{12} y^{-8}}{x^{-3} y^{12}} = \frac{x^{12} x^3}{y^{12} y^8} = \boxed{\frac{x^{15}}{y^{20}}}$$

Nov 15-7:05 AM

$$11) \left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

$$\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3 = 2^3 = \boxed{8}$$

$$\left(\frac{x^3}{5}\right)^{-2} = \left(\frac{5}{x^3}\right)^2 = \frac{5^2}{(x^3)^2} = \boxed{\frac{25}{x^6}}$$

$$\left(\frac{2x^4}{3y^8}\right)^{-4} = \left(\frac{3y^8}{2x^4}\right)^4 = \frac{(3y^8)^4}{(2x^4)^4} = \frac{3^4(y^8)^4}{2^4(x^4)^4}$$

$$= \boxed{\frac{81y^{32}}{16x^{16}}}$$

Nov 15-7:10 AM

Simplify

$$\left(\frac{-2x^{-6}}{5y^{-8}}\right)^{-3} = \left(\frac{-2y^8}{5x^6}\right)^{-3} = \left(\frac{5x^6}{-2y^8}\right)^3$$

$$= \frac{5^3(x^6)^3}{(-2)^3(y^8)^3} = \boxed{\frac{125x^{18}}{-8y^{24}}}$$

$$= \boxed{\frac{-125x^{18}}{8y^{24}}}$$

Nov 15-7:19 AM

Solve by graphing:

$$\begin{cases} 2x - y = 4 & \begin{array}{r|l} x & y \\ 0 & -4 \\ 2 & 0 \end{array} \\ 2x + 3y = 12 & \begin{array}{r|l} x & y \\ 0 & 4 \\ 6 & 0 \end{array} \end{cases}$$

System is consistent.
Eqns are independent.

Nov 15-8:14 AM

Solve by subs. method:

Hint: Isolate one variable

$$\begin{cases} 3x - y = 4 \\ -9x + 3y = -12 \end{cases}$$

$-y = -3x + 4$
 $y = 3x - 4$

$-9x + 3(3x - 4) = -12$

$\cancel{-9x} + \cancel{9x} - 12 = -12$

$0 = -12 + 12$
 $0 = 0$

True
infinitely many
Solns.
System \rightarrow Consistent
Eqns \rightarrow Dependent

Nov 15-8:19 AM

Solve by addition/elimination method

$$-2 \begin{cases} 2x + 4y = 5 \\ 4x + 8y = -9 \end{cases} \rightarrow \begin{cases} -4x - 8y = -10 \\ 4x + 8y = -9 \end{cases}$$

$$0 = -19$$

False

NO Solution

System \rightarrow inconsistent

Eqns \rightarrow Independent.

Nov 15-8:22 AM

Solve

$$3 \begin{cases} x - 3y = 7 \\ 4x + y = 5 \end{cases} \Rightarrow \begin{cases} x - 3y = 7 \\ 12x + 3y = 15 \end{cases}$$

$$13x = 22$$

$$x = \frac{22}{13}$$

$$-4 \begin{cases} x - 3y = 7 \\ 4x + y = 5 \end{cases}$$

$$\begin{cases} -4x + 12y = -28 \\ 4x + y = 5 \end{cases}$$

$$\hline 13y = -23$$

$$y = \frac{-23}{13}$$

$$\left(\frac{22}{13}, \frac{-23}{13} \right)$$

Nov 15-8:26 AM

Solve

$$\begin{cases} 6 \left\{ \frac{x}{2} - \frac{y}{3} = 9 \right. \\ 20 \left\{ \frac{x}{5} - \frac{y}{4} = 5 \right. \end{cases}$$

$$\begin{cases} 5 \left\{ 3x - 2y = 54 \right. \\ -2 \left\{ 4x - 5y = 100 \right. \end{cases}$$

Hint: Use LCD
For each eqn,
then clear
fractions.

$$\Rightarrow \begin{cases} 15x - 10y = 270 \\ -8x + 10y = -200 \end{cases}$$

$$7x = 70$$

$$3(10) - 2y = 54$$

$$-2y = 54 - 30$$

$$-2y = 24$$

$$y = -12$$

$$x = 10$$

$$(10, -12)$$

Nov 15-8:33 AM

41 Tickets Sold.

\$307 Collected

Kid's TKT \rightarrow \$5Adult's TKT \rightarrow \$11

use system of linear
equations to solve.

$$-5 \left\{ \begin{aligned} A + K &= 41 \\ 11A + 5K &= 307 \end{aligned} \right.$$

$$\begin{cases} -5A - 5K = -205 \\ 11A + 5K = 307 \end{cases}$$

$$6A = 102$$

$$6A = 102$$

$$A = 17$$

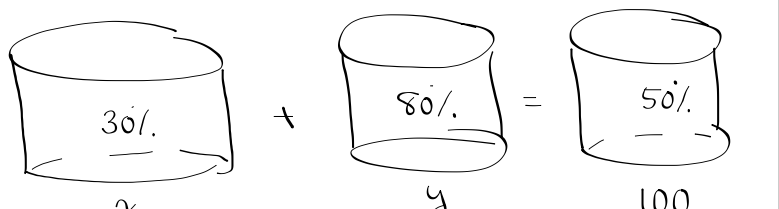
17 Adults

$$\&$$

24 Kids

Nov 15-8:41 AM

I need 100 liters of 50% acid solution.
I have unlimited supply of 30% & 80%
acid solutions. How many liters of each
should I mix to obtain what I need?



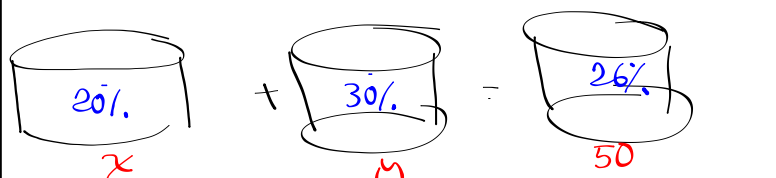
$$\begin{cases} x + y = 100 \\ 30\%x + 80\%y = 50\% \cdot 100 \end{cases} \Rightarrow \begin{cases} x + y = 100 \\ 30x + 80y = 50 \cdot 100 \end{cases}$$

$$\begin{cases} x + y = 100 \\ 3x + 8y = 500 \end{cases} \Rightarrow \begin{cases} -3x - 3y = -300 \\ 3x + 8y = 500 \end{cases} \Rightarrow \begin{cases} y = 40 \\ x = 60 \end{cases}$$

40L of 80%
60L of 30%

Nov 15-8:48 AM

I need 50 liters of 26% alcohol solution.
I have unlimited supply of 20% & 30%
alcohol solutions. How many liters of each?



$$\begin{cases} x + y = 50 \\ 20\%x + 30\%y = 26\% \cdot 50 \end{cases} \Rightarrow \begin{cases} x + y = 50 \\ 20x + 30y = 26 \cdot 50 \end{cases}$$

$$\begin{cases} x + y = 50 \\ 2x + 3y = 130 \end{cases} \Rightarrow \begin{cases} -2x - 2y = -100 \\ 2x + 3y = 130 \end{cases} \Rightarrow \begin{cases} y = 30 \\ x = 20 \end{cases}$$

30L of 30% soln
&
20L of 20% soln.

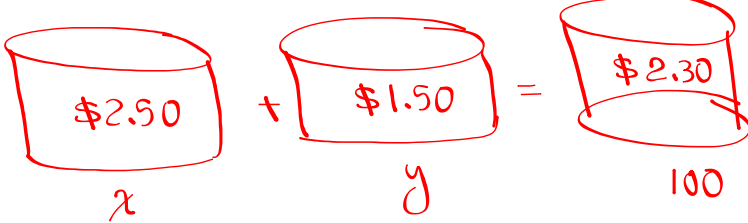
Nov 15-8:55 AM

Store has two brands of coffee.

Brand A sells for \$2.50/lb.

Brand B sells for \$1.50/lb.

The manager needs 100 pounds of coffee at \$2.30/lb. How many pounds of each should the manager mix to obtain that 100 pounds of coffee at \$2.30/lb.



$$\begin{cases} x + y = 100 \\ 2.50x + 1.50y = 2.30(100) \end{cases} \Rightarrow \begin{cases} x + y = 100 \\ 250x + 150y = 230(100) \end{cases}$$

Nov 15-9:05 AM

$$\begin{cases} x + y = 100 \\ 25x + 15y = 2300 \end{cases} \xrightarrow{-3} \begin{cases} x + y = 100 \\ 5x + 3y = 460 \end{cases}$$

$$\begin{cases} -3x - 3y = -300 \\ 5x + 3y = 460 \end{cases}$$

$$2x = 160$$

$$\boxed{x = 80}$$

80 lb. @ \$2.50/lb.

20 lb. @ \$1.50/lb.

Nov 15-9:11 AM

Annie Paid \$10.15 for 3 HB and 4 FF.

Raul Paid \$11.55 for 4 HB and 3 FF.

$$\begin{cases} -3 \{ 3H + 4F = 10.15 \\ 4 \{ 4H + 3F = 11.55 \end{cases} \rightarrow \text{we find Price for HB \& FF.}$$

$$\begin{cases} -9H - 12F = -30.45 \\ 16H + 12F = 46.20 \end{cases}$$

$$7H = 15.75$$

$$H = \frac{15.75}{7}$$

$$H = \$2.25$$

$$4(2.25) + 3F = 11.55$$

$$9 + 3F = 11.55$$

$$3F = 2.55$$

$$F = \$.85$$

$$HB \rightarrow \$2.25$$

$$FF \rightarrow \$.85$$

Nov 15-9:15 AM